[11:00-11:40] Review of midterm #2

Midterm #2 solutions

[11:50-12:00] Continuous time Fourier transform

Frequency domain analysis is one of the four pillars of the ECE curriculum.

Electrical engineering

1. Frequency domain: ECE 313 Linear Systems & Signals, 411 Circuit Theory, 302 Intro to Electrical Engineering, etc. (Called a characteristic function in ECE 351K)
2. Randomness: ECE 351K Probability + labs (measurement error and noise)

Computer engineering

3. State – finite of infinite: ECE 312, 306, 313
4. Complexity: ECE 312, 313, etc.

The **Fourier series (FS)** allow us to represent **periodic** signals in the frequency domain. The **continuous time Fourier transform (FT)** allows us to represent **periodic and aperiodic** signals in the frequency domain. The Fourier transform provides a general definition of the frequency spectrum.

Fourier transform analysis (using a frequency variable ω in radians per second):

$$\mathcal{F}\{x(t)\} = X(j\omega) = \int_{t=-\infty}^{\infty} x(t)e^{-j\omega t}\, dt$$

Synthesis (inverse transform):

$$\mathcal{F}^{-1}\{X(j\omega)\} = x(t) = \frac{1}{2\pi}\int_{t=-\infty}^{\infty} X(j\omega)e^{j\omega t}\, dt$$

Examples of continuous-time Fourier transform pairs:

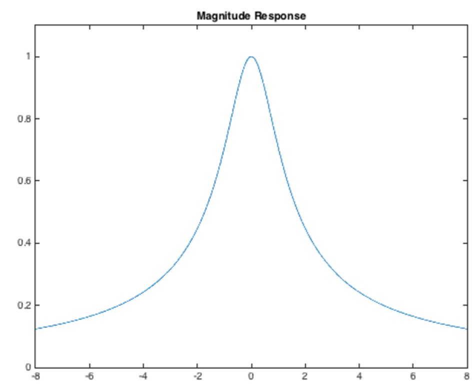
$$\mathcal{F}\{\delta(t)\} = 1$$

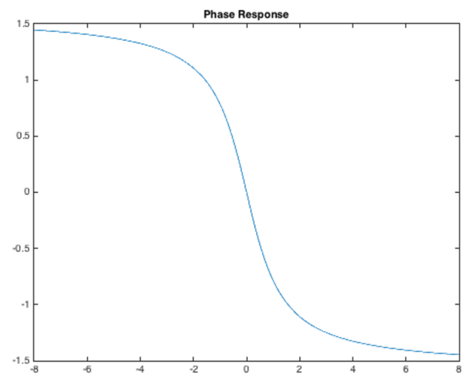
$$\mathcal{F}\{\delta(t-T)\} = e^{-j\omega T}$$

[12:00] Fourier transform of causal exponential signal

$$x(t) = e^{-at}u(t)$$

$$\mathcal{F}\{x(t)\} = \int_{t=-\infty}^{\infty} x(t)e^{-j\omega t}\,dt = \int_{t=-\infty}^{\infty} e^{-at}u(t)e^{-j\omega t}\,dt$$
$$= \int_{t=0}^{\infty} e^{-at}e^{-j\omega t}\,dt = \int_{t=0}^{\infty} e^{-a-j\omega t}\,dt$$
$$= \left.\frac{e^{-a-j\omega t}}{-a-j\omega}\right|_{0}^{\infty} = \begin{cases} \dfrac{1}{a+j\omega} & Re\{a\} > 0 \\ \text{integral does not converge} & \text{otherwise} \end{cases}$$



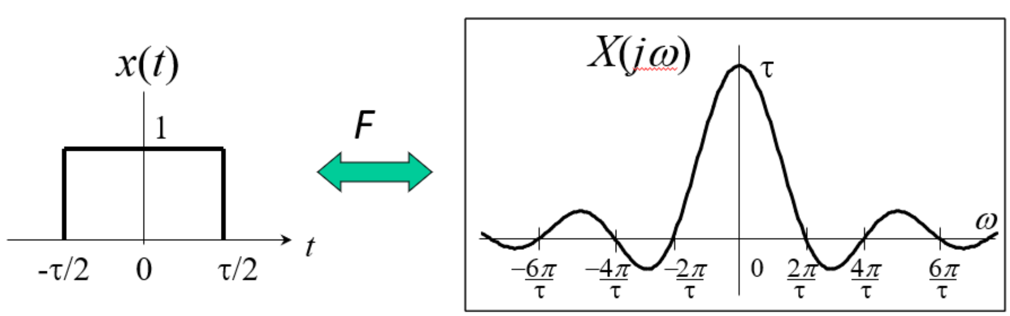


[12:05] Fourier transform of rectangular pulse

$$\mathcal{F}\left\{rect\left(\frac{t}{\tau}\right)\right\} = \int_{t=-\infty}^{\infty} rect\left(\frac{t}{\tau}\right)e^{-j\omega t}\,dt$$

$$= \int_{t=-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{-j\omega t}\,dt = -\frac{1}{j\omega}\left(e^{-\frac{j\omega\tau}{2}} - e^{j\omega\tau}\right)$$

$$= \tau\frac{\sin\left(\pi\frac{\omega\tau}{2\pi}\right)}{\pi\left(\frac{\omega\tau}{2\pi}\right)} = \tau\,\text{sinc}\left(\frac{\omega\tau}{2\pi}\right)$$

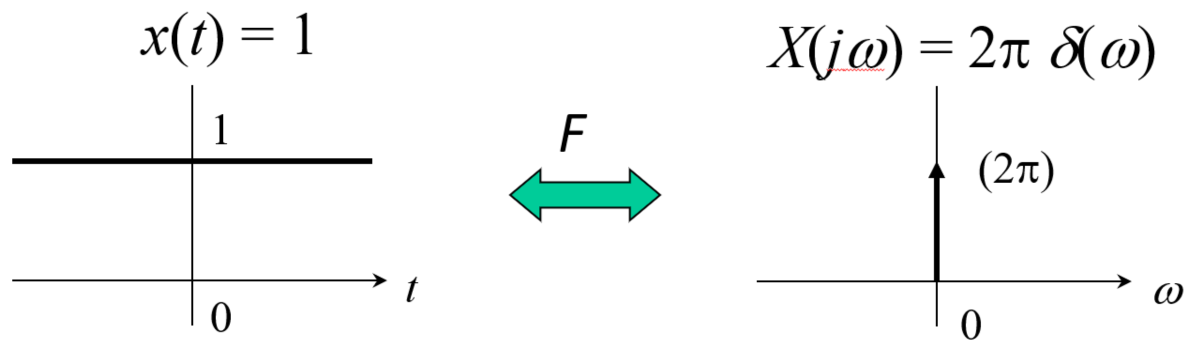


[12:10] Impulse in time or frequency

$$\mathcal{F}\{\delta(t)\} = 1$$

$$\mathcal{F}^{-1}\{\delta(\omega)\} = \frac{1}{2\pi}$$

$$\mathcal{F}\{1\} = 2\pi\delta(\omega)$$



[12:15] Fourier transform of two-sided cosine

$$\mathcal{F}\{\cos(\omega_0 t)\} = \pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$$